

EE 135.Winter 2013. Laboratory 2. Electrostatics: Resistive Paper Analog

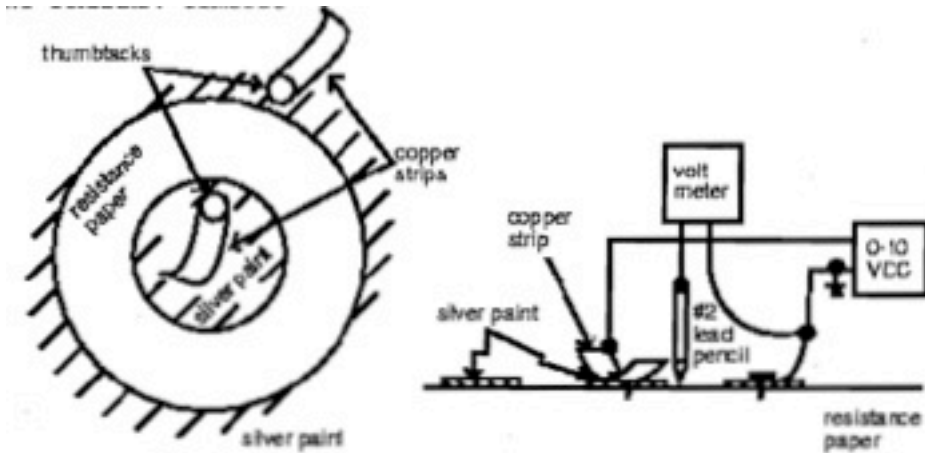
Introduction:

It is shown that the Laplace's equation ($\nabla^2 V = 0$), which governs the electrostatic potential distribution, with certain boundary conditions is satisfied for steady current flows in a resistive medium. (See Sec. 1.13 of reference [1].

Since $\vec{J} = \sigma \vec{E} = -\sigma \nabla V$, and continuity demands $\nabla \cdot \vec{J} = -\frac{d\rho}{dt} = 0$, Laplace's

equation is obeyed and J , E are in the same direction. The equipotential lines are perpendicular to the current lines.) Therefore, a resistive sheet can be used as an analog to plot the equipotentials and electric field lines, and to determine the capacitance, for a two-dimensional electrostatic field configuration. The analog is also related to graphical field mapping techniques as described in Sec. 1.19-1.20 of reference [1].

The experiment apparatus is shown in the figure below. Silver paint is conducting. It is used to determine certain boundary conditions. The thumbtacks together with the copper strips are connected to the anode and cathode of a voltage source, which then will generate a steady current flow across the resistive paper. The equipotentials are measured by a voltmeter through a lead pencil, which will trace out the equipotential lines on the resistive paper.



Procedure:

- Using the " Ω 2W" function on a digital multimeter, measure the resistance of a square (approx. 4") of resistive paper coated with silver paint at its two opposite edges. $R_{sq} = \underline{\hspace{2cm}} \Omega$.

Cut this square to 2" square and measure the resistance again.

$$R_{sq} = \underline{\hspace{2cm}} \Omega$$

Does the measured value of R_{sq} depend on the size of the square? Why?

 (Y/N)

- Coaxial line

Place template #1 (metal) onto the center of a large piece of resistive paper. Carefully paint the silver paint around the inside of the inner circle (representing the inner conductor of a coaxial line) and around the outside of the outer circle (representing the outer conductor of a coaxial line). Use the fan to dry the paint.

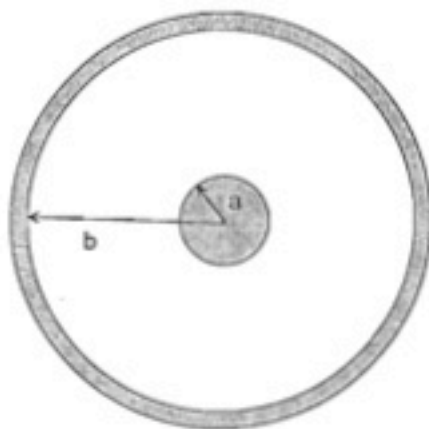
Use thumbtacks to attach copper strips to the inner and outer "conductors". Apply 10 V between the inner (+) and outer (- or GND) conductor using the voltage source.

Using the "DC V" function on the digital multimeter with lead pencil probe, find and mark the 5 V equipotential line down onto the resistive paper. Is this equipotential line a circle? _____ (Y/N)

Is it midway between the inner and outer conductors? _____ (Y/N)

Why? (hint: $V \propto \ln \frac{a}{r}$)

Find and mark down the 2 V and 8 V equipotential lines as well. Sketch all the equipotential lines you have found below. Label the voltage for each line.



Disconnect the voltage source first! Then measure the resistance between the inner and outer electrodes using an ohmmeter.

$R_{\text{COAX}} = \text{_____} \Omega$

Cut a small square piece from the large resistive paper you are using and measure the resistance. $R_{\text{SQ}} = \text{_____} \Omega$

It is shown that the capacitance per unit length of a coaxial line in this measurement is given by $C = \epsilon \frac{R_{\text{SQ}}}{R_{\text{COAX}}}$ (see Sec. 1.21 of reference [1]).

Given $\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m, calculate capacitance from your measurement.

$$C_{\text{meas}} = \text{_____ F/m}$$

In the text, Table 2-1, the capacitance per unit length is given by

$$C = \frac{2\pi\epsilon}{\ln(b/a)}, \text{ where } a \text{ and } b \text{ are the radii of the inner and the outer}$$

conductors ($a = 2.5$ cm, $b = 7$ cm). Please calculate the capacitance using this formula and compare it with the value you obtained in the previous step. $C_{\text{calc}} = \text{_____ F/m}$

3. Two-wire transmission line

Place template #2 (metal) onto the center of another large piece of resistive paper. Paint *only* the two circles (representing two wires). Use thumbtacks to attach copper strips to the two silver-painted circles.

Apply 10 V between the two wires (+ on the right circle). Plot equipotential lines at 5 V, 2 V and 8 V on the resistive paper, and sketch them below.



Turn off the voltage source and then measure the resistance between the two wires. $R_{\text{wire}} = \text{_____ } \Omega$

Cut a small square piece from the large resistive paper you are using and measure the resistance $R_{\text{sq}} = \text{_____ } \Omega$.

Determine C from your measurements using the same equation you used previously. $C_{\text{meas}} = \underline{\hspace{2cm}}$ F/m

Compare with C from Table 2-1 in the text, where a is the radius of the wire and d is the distance between two wires. $a = 1.25$ cm, $d = 7$ cm.

$$C_{\text{wire}} = \frac{\pi\epsilon}{\ln\left[\frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right]} = \underline{\hspace{2cm}} \text{ F/m}$$

4. Image Method (see text Sec. 4-12 or ref. [1] Sec. 1.18)

Paint the line on template #2 between the two previously painted circles and then paint the right side of the line with silver paint. Apply 5 V between the left circle and the conducting plane on the right (5 V on the plane and GND on the circle). Replot the 2 V equipotential. Does the method of images yield the same equipotential as found in the two-wire line?

Ref.1 Fields and Waves in Communication Electronics. Ramo, Whinnery and Van Duzer. John Wiley and Sons. Or Chapter 1,2, 3 of the class text.